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PRECISE AND EFFICIENT COMPUTATION OF COMPLEX STRUCTURES WITH TMD DEVICES

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Proportionally damped structures give non-proportionally damped mathematical models when tuned mass damper (TMD) devices and/or viscous dampers are installed. The precise analysis of unwanted enforced vibration of such a combined system, i.e., of the structure plus the TMD devices and/or dampers, is very cumbersome and inefficient for structures with many (e.g., 10^3–10^5) degrees of freedom. This short paper gives a convenient and efficient method for computing the results for such problems which involves only the first few undamped modes of the structure and is not iterative. Results are given for a simple illustrative truss and for an actual offshore jacket platform.

1. INTRODUCTION

Tuned mass damper (TMD) devices are widely used for suppressing unwanted enforced vibration of engineering structures, e.g., long-span bridges, high rise buildings and rotating machines. Although many related publications are available [e.g., 1–7], a good solution is not yet available for the following very basic problem.

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If the finite element method (FEM) model of the structure has many degrees of freedom (DOF), e.g., 10^3–10^5, the equations of motion are usually written on the assumption of proportional damping before attachment of TMD devices or dampers in order to use the mode superposition scheme. However, when TMD devices and/or dampers are attached to that structure, their damping coefficient will not be the same as that of the structure and so the resulting equations of the combined system are non-proportionally damped. Hence, although well-known programs, e.g., SAP and ANSYS, can solve harmonic enforced vibration for proportionally damped structures with many DOF very easily, the installation of TMD devices and/or dampers on such structures makes the accurate analysis of their reduced amplitude vibration very much harder. The conventional method for computing accurately the responses of such a combined system is to use its complex modes to decouple the corresponding equations of motion, which is feasible only for relatively simple structures because of the high CPU time and storage space requirements.

An alternative approach [8] uses the undamped modes of the original structure to reduce the equations of motion of the combined system. This gives equations with far fewer DOF, but the damping matrix is non-diagonal, so that the equations are coupled and hence can only be solved by a step-by-step integration method. Obviously, this alternative is not particularly straightforward and gives results that are not accurate unless a long time history is used for the computation.

Reference [6] gives an iterative algorithm for dealing with such non-proportionally damped structures, but it has been proved [7] that this algorithm diverges under some circumstances.

The present paper develops a new computational method for solving such non-proportionally damped problems. This method uses only the undamped modes of the original structure to reduce the equations of motion of the combined system and hence to enable the solution to be computed quickly. The method is direct, i.e., no iteration is required, and so the problem of divergence does not arise. Hence the method is convenient and precise.

2. BASIC PRINCIPLE AND ALGORITHM

One assumes that the equations of motion of the discretised original structure are

$$[\mathbf{M}_o]\{\ddot{\mathbf{y}}_o\} + [\mathbf{C}_o]\{\dot{\mathbf{y}}_o\} + [\mathbf{K}_o]\{\mathbf{y}_o\} = \{\mathbf{F}_o\} = \{\mathbf{P}\}\exp(\mathrm{i}\theta t), \tag{1}$$

in which $[\mathbf{M}_o]$, $[\mathbf{C}_o]$ and $[\mathbf{K}_o]$ are the $n \times n$ mass, damping and stiffness matrices of the original structure, respectively, $\{\mathbf{y}_o\}$ is its displacement vector and $\{\mathbf{F}_o\}$ is the sinusoidal loading vector, which has amplitude vector $\{\mathbf{P}\}$ and angular frequency θ, where $\mathrm{i} = \sqrt{-1}$. The first q normalised real modes comprise the $n \times q$ matrix $[\mathbf{\Phi}_o]$ ($q \ll n$), which satisfies the equations

$$[\mathbf{\Phi}_o]^{\mathrm{T}}[\mathbf{M}_o][\mathbf{\Phi}_o] = [\mathbf{I}]_q, \qquad [\mathbf{\Phi}_o]^{\mathrm{T}}[\mathbf{C}_o][\mathbf{\Phi}_o] = [\mathbf{C}_o]^* = [2\zeta_j\omega_j], \tag{2, 3}$$

$$[\mathbf{\Phi}_o]^{\mathrm{T}}[\mathbf{K}_o][\mathbf{\Phi}_o] = [\mathbf{\Lambda}_o^2], \qquad \{\mathbf{F}_o\}^* = [\mathbf{\Phi}_o]^{\mathrm{T}}\{\mathbf{F}_o\} = [\mathbf{\Phi}_o]^{\mathrm{T}}\{\mathbf{P}\}\exp(\mathrm{i}\theta t), \tag{4, 5}$$

where $[\mathbf{I}]_q$ is the $q \times q$ unit matrix, $[2\zeta_j\omega_j]$ is a diagonal matrix with $2\zeta_j\omega_j$ as its jth diagonal element, ζ_j and ω_j are the damping ratio and angular frequency associated with the jth real mode and $[\mathbf{\Lambda}_o^2]$ is the $q \times q$ diagonal matrix with ω_j^2 as its jth diagonal element.

If m TMD devices and r isolated viscous dampers are used, the combined system will have m more DOF than the original structure. Suppose that the assembled stiffness, mass and damping matrices of all of these m TMD devices and r dampers are, in the structural co-ordinate system

$$[\mathbf{K}]^D = \begin{bmatrix} \mathbf{K}_{ss} & \mathbf{K}_{sT} \\ \mathbf{K}_{Ts} & \mathbf{K}_{TT} \end{bmatrix}, \qquad [\mathbf{M}]^D = \begin{bmatrix} \mathbf{M}_{ss} & 0 \\ 0 & \mathbf{M}_{TT} \end{bmatrix}, \qquad [\mathbf{C}]^D = \begin{bmatrix} \mathbf{C}_{ss} + \mathbf{C}_{ss}^D & \mathbf{C}_{sT} \\ \mathbf{C}_{Ts} & \mathbf{C}_{TT} \end{bmatrix}, \quad (6)$$

in which subscript s refers to the DOF of the original structure, subscript T refers to the added DOF for all m TMD devices and $\mathbf{C}_{ss}^D$ represents the contribution of all r isolated dampers, which can be regarded as degenerated TMD systems that add no extra DOF to the system. Hence the rows and columns of matrices $[\mathbf{K}]^D$, $[\mathbf{M}]^D$ and $[\mathbf{C}]^D$ that correspond to those of the original structure have been added to by rows and columns corresponding to the TMD devices, and all three will therefore be $(n+m) \times (n+m)$ matrices. Thus, the combined system has the global stiffness, mass and damping matrices

$$[\mathbf{K}] = \begin{bmatrix} \mathbf{K}_o + \mathbf{K}_{ss} & \mathbf{K}_{sT} \\ \mathbf{K}_{Ts} & \mathbf{K}_{TT} \end{bmatrix}, \qquad [\mathbf{M}] = \begin{bmatrix} \mathbf{M}_o + \mathbf{M}_{ss} & 0 \\ 0 & \mathbf{M}_{TT} \end{bmatrix},$$

$$[\mathbf{C}] = \begin{bmatrix} \mathbf{C}_o + \mathbf{C}_{ss} + \mathbf{C}_{ss}^D & \mathbf{C}_{sT} \\ \mathbf{C}_{Ts} & \mathbf{C}_{TT} \end{bmatrix}, \quad (7)$$

and the corresponding $(n+m)$ element displacement vector is

$$\{\mathbf{y}\} = \begin{Bmatrix} \mathbf{y}_o \\ \mathbf{y}_T \end{Bmatrix}, \quad (8)$$

in which the m dimensional displacement vector $\{\mathbf{y}_T\}$ represents the displacements of the masses of the TMD devices.

Now, by using equations (2)–(5) and introducing the co-ordinate transformation matrix

$$[\mathbf{\Phi}] = \begin{bmatrix} \mathbf{\Phi}_o & 0 \\ 0 & \mathbf{I}_m \end{bmatrix}, \quad (9)$$

in which $[\mathbf{I}_m]$ is an $m \times m$ unit matrix, equation (7) can be reduced to

$$[\mathbf{K}]^* = [\mathbf{\Phi}]^{\mathrm{T}}[\mathbf{K}][\mathbf{\Phi}] = \begin{bmatrix} \mathbf{\Lambda}_o^2 + \mathbf{\Phi}_o^{\mathrm{T}}\mathbf{K}_{ss}\mathbf{\Phi}_o & \mathbf{\Phi}_o^{\mathrm{T}}\mathbf{K}_{sT} \\ \mathbf{K}_{Ts}\mathbf{\Phi}_o & \mathbf{K}_{TT} \end{bmatrix}, \quad (10)$$

$$[\mathbf{M}]^* = [\mathbf{\Phi}]^{\mathrm{T}}[\mathbf{M}][\mathbf{\Phi}] = \begin{bmatrix} \mathbf{I}_q + \mathbf{\Phi}_o^{\mathrm{T}}\mathbf{M}_{ss}\mathbf{\Phi}_o & 0 \\ 0 & \mathbf{M}_{TT} \end{bmatrix}, \tag{11}$$

$$[\mathbf{C}]^* = [\mathbf{\Phi}]^{\mathrm{T}}[\mathbf{C}][\mathbf{\Phi}] = \begin{bmatrix} \mathbf{C}_o^* + \mathbf{\Phi}_o^{\mathrm{T}}\mathbf{C}_{ss}\mathbf{\Phi}_o + \mathbf{\Phi}_o^{\mathrm{T}}\mathbf{C}_{ss}^{D}\mathbf{\Phi}_o & \mathbf{\Phi}_o^{\mathrm{T}}\mathbf{C}_{sT} \\ \mathbf{C}_{Ts}\mathbf{\Phi}_o & \mathbf{C}_{TT} \end{bmatrix}, \tag{12}$$

and the corresponding loading vector becomes

$$\{\mathbf{F}\}^* = \begin{bmatrix} \mathbf{\Phi}_o^{\mathrm{T}} & 0 \\ 0 & \mathbf{I}_m \end{bmatrix} \begin{Bmatrix} \mathbf{F}_o \\ 0 \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_o^* \\ 0 \end{Bmatrix} = \begin{Bmatrix} [\mathbf{\Phi}_o]^{\mathrm{T}}\{\mathbf{P}\} \\ 0 \end{Bmatrix} \exp(\mathrm{i}\theta t) \tag{13}$$

Now $\{\mathbf{y}\}$, the displacement vector of the combined system before the DOF reduction, can be expressed in terms of $\{\mathbf{z}\}$, the displacement vector after the DOF reduction associated with equation (9), by

$$\{\mathbf{y}\} = [\mathbf{\Phi}]\{\mathbf{z}\} \tag{14}$$

and the equations of motion after this DOF reduction are

$$[\mathbf{M}]^*\{\ddot{\mathbf{z}}\} + [\mathbf{C}]^*\{\dot{\mathbf{z}}\} + [\mathbf{K}]^*\{\mathbf{z}\} = \{\mathbf{F}\}^*. \tag{15}$$

In general, these equations are not proportionally damped, and so are coupled. However, since the loadings are sinusoidal, they can be expressed as

$$\{\mathbf{F}\}^* = \{\mathbf{Q}\}\exp(\mathrm{i}\theta t), \tag{16}$$

in which

$$\{\mathbf{Q}\} = \begin{Bmatrix} [\mathbf{\Phi}_o]^{\mathrm{T}}\{\mathbf{P}\} \\ 0 \end{Bmatrix} \tag{17}$$

and the solution to equations (15) can be expressed as

$$\{\mathbf{z}\} = [\{\mathbf{A}\} + \mathrm{i}\{\mathbf{B}\}]\exp(\mathrm{i}\theta t), \tag{18}$$

where $\{\mathbf{A}\}$ and $\{\mathbf{B}\}$ are two real constant vectors to be determined. Therefore substituting equation (18), its derivatives $\{\dot{\mathbf{z}}\} = \mathrm{i}\theta\{\mathbf{z}\}$ and $\{\ddot{\mathbf{z}}\} = -\theta^2\{\mathbf{z}\}$, plus equation (16), into equations (15) gives

$$(-\theta^2[\mathbf{M}]^* + \mathrm{i}\theta[\mathbf{C}]^* + [\mathbf{K}]^*)[\{\mathbf{A}\} + \mathrm{i}\{\mathbf{B}\}] = \{\mathbf{Q}\}. \tag{19}$$

Finally, expanding the left side and comparing its real and imaginary parts gives

$$[\mathbf{E}]\{\mathbf{A}\} + [\mathbf{D}]\{\mathbf{B}\} = \{\mathbf{Q}\}, \qquad -[\mathbf{D}]\{\mathbf{A}\} + [\mathbf{E}]\{\mathbf{B}\} = \{\mathbf{0}\}, \tag{20}$$

in which

$$[\mathbf{E}] = [\mathbf{K}]^* - \theta^2[\mathbf{M}]^*, \qquad [\mathbf{D}] = -\theta[\mathbf{C}]^*. \tag{21}$$

It is very easy to compute solutions for $\{\mathbf{A}\}$ and $\{\mathbf{B}\}$ from the $2q$ linear algebraic equations (20) and then to compute $\{\mathbf{z}\}$ and $\{\mathbf{y}\}$ from equations (18) and (14). Note that in the above derivation the only assumption made is that only a limited number of modes (i.e., q) are taken for the mode superposition, so that the method may be regarded as an accurate direct method.

3. COMPUTATION PROCEDURE

The computation procedure corresponding to the derivation described above can be briefly summarised as follows:

(1) Compute the classical (real) modes $[\mathbf{\Phi}_o]$ and the corresponding eigenvalues $[\mathbf{\Lambda}_o^2]$ of the original structure, then use equation (9) to generate $[\mathbf{\Phi}]$.
(2) Use equation (6) to produce the assembled stiffness, mass and damping matrices $[\mathbf{K}]^p$, $[\mathbf{M}]^p$ and $[\mathbf{C}]^p$ of all TMD devices and/or dampers.
(3) Use equations (10)–(12) to produce the stiffness, mass and damping matrices for the combined system.
(4) Use equations (17) and (21) to compute vector $\{\mathbf{Q}\}$ and matrices $[\mathbf{E}]$ and $[\mathbf{D}]$.
(5) Solve equations (20) for $\{\mathbf{A}\}$ and $\{\mathbf{B}\}$.
(6) Substitute $\{\mathbf{A}\}$ and $\{\mathbf{B}\}$ into equations (18) and (14) to find the displacements $\{\mathbf{y}\}$ for the system with TMD devices and/or isolated dampers.
(7) If necessary, compute the internal forces and/or any other quantities of interest from $\{\mathbf{y}\}$.

4. NUMERICAL EXAMPLE

Two examples are given. The first is sufficiently simple for it to be fully specified, so that it can be repeated and/or used as a comparator by other authors, whereas the second is a major real-life structure for which it is reasonable only to specify the main items of data.

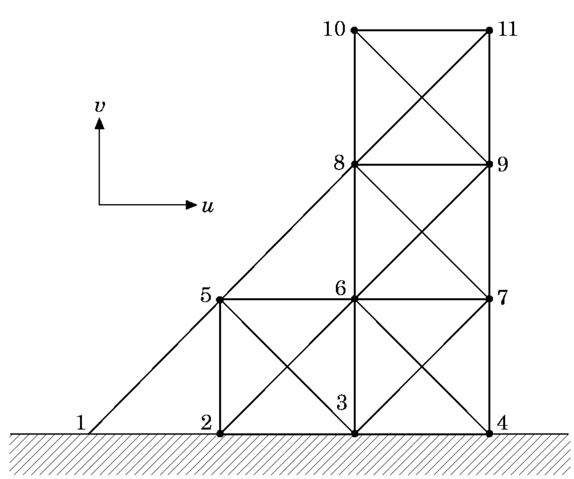

Figure 1. 21 member plane truss.

TABLE 1

Horizontal (u_{11}) *and vertical* (v_{11}) *displacement amplitudes for joint* 11 *obtained using the first five natural angular frequencies* ($q = 5$)

Numbers and location of TMD devices used				
Number	Between nodes	At nodes	u_{11}(mm)	v_{11}(mm)
0	–	–	41·78	15·81
2	10–11, 8–11	–	32·50	12·28
2	10–11, 9–10	–	30·09	11·57
2	10–11, 8–9	–	31·89	12·13
2	9–10, 8–11	–	39·57	15·02
3	9–10, 8–11, 10–11	–	28·58	10·91
4	9–10, 8–11, 10–11, 8–9	–	26·60	10·15
2	–	9, 11	40·74	15·42
2	–	8, 10	40·70	15·41

The first example is shown in Figure 1. It consists of a 21 element plane truss which is located on a shaking table which imposes a horizontal acceleration of 1·0 exp($i\theta t$) m/s^2, with $\theta = 16{\cdot}9$ (1/s). The length of each horizontal or vertical member is 5 m, the axial rigidity of every member is 3×10^7 kN and all members are massless. There is a lumped mass of 1000 kg at every node and hence its first five natural angular frequencies are 16·92, 40·75, 46·41, 78·88 and 94·67 (1/s). TMD devices with mass 100·0 kg, stiffness 28561·0 N/m and damping coefficient 100·0 kg/s are used to reduce the enforced structural vibration.

Table 1 shows the horizontal and vertical displacement amplitudes at node 11 when no TMD devices are installed (as a basis for comparison) and for eight alternative configurations of up to four TMD devices (see Figure 2). These devices were installed between joints, except for the final two cases, for which they were attached horizontally at the joints. It can be seen that the four TMD devices case gave the best results, reducing u_{11} by $\{(4{\cdot}178 - 2{\cdot}660)/4{\cdot}178\} \times 100\% = 36{\cdot}3\%$ and v_{11} by 35·8%.

The second example is the jacket platform of Figure 3 which has been built in the Bohai sea of northern China. Its height is 40 m and it has a square base with side length 19·07 m The 3D FEM model used had 86 nodes, 183 beam elements and 492 DOF and gave its lowest eight natural angular frequencies as 3·8325, 3·8811, 4·1619, 7·9105, 9·9539, 11·205, 11·547 and 12·091 (1/s). The damping ratio used for all participating modes was 0·03 and it was assumed that two parallel in-phase sinusoidal excitations P exp($i\theta t$) acted at nodes 5 and 11, in the direction 5–9, with $P = 10$ kN and $\theta = 3{\cdot}85$ 1/s. TMD devices and/or viscous

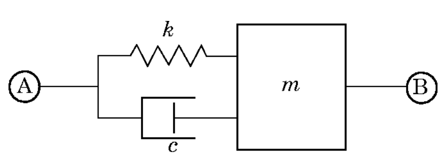

Figure 2. Schematic diagram of a TMD device between two nodes A and B.

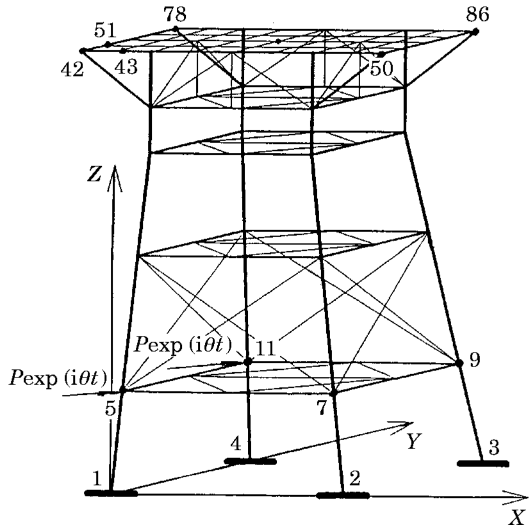

Figure 3. Jacket platform.

dampers were optionally present, to give the eight cases of Table 2, in which the second and third columns give the number of TMD devices, N_T, and the number of dampers, N_D. The values of ΔM and ΔK given in the fourth and fifth columns are the mass and stiffness of the TMD device, respectively, and ΔC in the sixth column is the damping coefficient of each TMD device or viscous damper. The seventh column usually shows, on each side of the dash, the numbers of a pair of nodes connected by one of the TMD devices or viscous dampers. However, for case 3 the number of a node and the alignment of a TMD device connected to it are shown on either side of the dash, using the X and Y directions shown on Figure 3. Note that case 1 is the original structure with no TMD devices or dampers and is included as a comparator for the other cases.

TABLE 2

Displacement amplitudes at node 86 *and axial force amplitude of members* 1–5 *obtained using the first eight natural angular frequencies* ($q = 8$)

(1) CASE	(2) N_T	(3) N_D	(4) ΔM (kg)	(5) ΔK (N/m)	(6) ΔC (kg/s)	(7) Installation type	(8) u_{86} (mm)	(9) v_{86} (mm)	(10) N_{1-5} (kN)
1	0	0	–	–	–	none	45·38	51·34	44·00
2	2	0	1000	14 800	380	42–43, 42–51	3·380	3·912	2·240
3	2	0	1000	14 800	380	42–X, 42–Y	9·066	11·01	9·018
4	2	0	1000	14 800	0	42–43, 42–51	3·375	3·923	2·243
5	0	2	–	–	380	42–43, 42–51	45·38	51·34	44·00
6	0	2	–	–	380	1–9, 4–7	43·72	49·33	42·28
7	0	2	–	–	38 000	1–9, 4–7	13·44	20·66	8·457
8	2	2	CASE (3) + CASE (6)				9·037	10·88	8·924

Columns (8)–(10) of Table 2 give the computed values of u_{86} and v_{86}, the amplitudes of the X and Y direction displacements at node 86, and of N_{1-5}, the amplitude of the axial force of the member connecting nodes 1 and 5. Case 1 shows that very strong resonant responses are caused for the original structure, because the lowest two frequencies of the structure are very close together and the excitation frequency $\theta = 3{\cdot}85\,(1/\mathrm{s})$ lies between them. When two TMD devices with natural angular frequency $\sqrt{\Delta K/\Delta M} = 3{\cdot}87\ (1/\mathrm{s}) \approx \theta$ were installed on the top deck, cases 2 and 3 show that very satisfactory results were obtained, but with devices connecting two pairs of nodes, i.e., case 2, being substantially better than two devices attached to one node of the deck, i.e., case 3. Furthermore, when the dampers in the TMD devices of case 2 were both removed, it had very little effect, see case 4. Note that the damping ratio $\Delta C/(2\sqrt{\Delta M \Delta K}) = 0{\cdot}05$ of these TMD systems was very small.

Case 5 shows that installation of two isolated dampers on the top deck with ΔC equal to that of the devices of cases 2 and 3, was entirely useless for suppressing the vibration. Case 6 shows that this remained almost true even when the two dampers were installed between nodes 1 and 9 and between nodes 4 and 7, where they were subjected to much larger strains, unless their damping coefficient is increased considerably, e.g., case 7 shows quite a good effect when it was increased by a factor of 100. Finally, case 8 shows that if the two TMD devices of case 3 and the two dampers of case 6 were installed simultaneously, the effect was only very marginally better than for case 3, i.e., for the TMD devices alone.

5. CONCLUSIONS

The problem of selecting appropriate values for the parameters of TMD devices has not been addressed because it has been discussed extensively and comprehensively elsewhere. Instead, the purpose of this article was to present an accurate and efficient algorithm for structures which are non-proportionally damped due to the installation of TMD devices and/or isolated dampers. The results presented show the method gives reasonable results.

Although rather straightforward, the method is of significance because of the wide range of practical engineering problems to which it can be applied. For example, the results of Table 2 are for a large offshore platform and indicate where TMD devices should be attached for best effect.

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